Section B

Algebraic expressions with one and two variables, Inequalities, Absolute value, and functions are the main topics covered in this section of quantitative review. Not so difficult algebra has been included on the GAT that it becomes a rock to crack. I have explained many of the simple rules to have the right answers without employing deep mathematical concepts.

Algebra

The methods or rules used in algebra are actually the extension of the methods or rules used in arithmetic. In algebra we use letters and symbols with or without numbers to represent quantities for example, in algebra we have terms like x^2, $5ab$, $-3a^2xy$ etc. Consider a term ($-3a^2xy$) used in algebra. It has following parts.

Coefficient:

The number part of this term i.e.(-3) is called coefficient.

Variable:

The letters or symbols used in an algebraic expression are called variables. 'a', 'x' and 'y' are three distinct variables.

Value of a Variable:

A variable may have one or more values in a given situation.

If $x^2 = 4$, then x may have two values (2) and (-2). If a condition that x is positive number is applied, then the value of x is only one that is (2).

Value of a variable can be changed according to the situation. That's why they are called variables.

Power or Exponent:

In $4x^2$, 2 is power or the exponent of x. in algebra. In general:

$a^n \times a^m = a^{m+n}$

$a^n \times a^m = a^{n+m}$

$(a^n)^m = a^{nm}$

$A^m \div A^n = A^{m-n}$

$A^n \div A^m = A^{n-m}$

If a number or variable has zero as an exponent, it is always equal to 1. i.e., $X^0 = 1$

Fractions in Power:

Same rules of exponents are applied for an algebraic expression if it has power in fraction.

$\sqrt{x} = x^{\frac{1}{2}}$

$\sqrt[3]{x} = x^{\frac{1}{3}}$

$\sqrt{x^2} = \pm x$

The solution of $\sqrt{-x^2}$ is out of scope of GAT's quantitative.

$\sqrt{x} \times \sqrt{x} = x$

$\sqrt[3]{x} \times \sqrt[3]{x} \times \sqrt[3]{x} = x$

$\sqrt{x^3} = +x$ and $\sqrt{-x^3} = -x$

$\frac{x}{\sqrt{x}} = \sqrt{x}$

$\frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}$

Surds

Expressions such as $\sqrt{2}, \sqrt{3}, \sqrt{7}$ cannot be written as numerically exact quantities. Such numbers are called irrational or surds. For example $\sqrt{72}$ in the simplest possible surd can be written as $\sqrt{72} = \sqrt{(36 \times 2)} = 6\sqrt{2}$

Constants

All numbers used in an algebraic expression either as exponents, as coefficients or as a term are called constants as they have a fixed value.

Algebraic Expressions:

All expressions that connect variables, constants (coefficients) etc by algebraic operations of addition, subtraction, multiplication, and division are called algebraic expressions for example, $2xy + y$, $y^3 + y^9$, $Z^3 - 5ab,c + \sqrt{d}$ + etc.

Algebraic Term

An algebraic expression is a combination of algebraic terms joined by mathematical operations '+' or '-' Thus each part of an algebraic expression separated by '+'or '-'sign is an algebraic term or simply a term; for example, in expression $(x^3 - 3xy + 2xy^2 + 12)$ there are four terms (x^3, 3xy, $2xy^2$ and 12).

Like Term

The algebraic terms having exactly the same variables and exponents are called like terms. They can differ only in their coefficients; for example, $(5x^2)$ and $(2x^2)$ are like terms whereas $(5x^2)$ and $(5y^2)$ are not.

Monomial

An algebraic expression that has only one term is called monomial. For example, 7, x^2, $2y^3$, $-4xy^3$, a^2b^2etc.

Binomial

An algebraic expression having only two terms is called binomial. For example = $x^2 + y^2$, $2x^4 + 12$ and $3x^3 - 9$ etc.

Trinomials

An algebraic expression with three terms is called trinomials. For example, $3x^2 + 2y + 2$, $3x^2 + 15x - 1$ and $7x^2 - 2xy + 2x^2y^6$ etc.

Polynomial

An algebraic expression with more than three terms is called polynomial. In general, binomials and trinomials are also included or called polynomial. For example: $3x^2 + 53$, $5x^6 + 7$, $15x^2 - 2xyz + 7xy^2 + 1$

Operations on Algebraic Expressions

All mathematical operations can be applied to the algebraic expressions. Following is the detail of how to perform these operations.

PEMDAS- Sequence of Operations

The order or precedence of operations for algebraic expressions is same as that for numbers, we have discussed earlier in numbers.

Addition

In addition of algebraic expressions, coefficients of like terms of both expressions are added; for example:

$(2x^2 + 3x + 5) + (3x^2 + 4x + 7) =$

$2x^2$	$3x$	5
$3x^2$	$4x$	7
$5x^2$	$7x$	12

Subtractions

In subtraction the signs of all the terms of expression that is to be subtracted are inverted i.e.'+' to '-'and '-'to '+' then coefficients of like terms are added; for example

$(2x^2 + 8x + 5) - (3x^2 + 4x + 2) =$

$2x^2$	$8x$	5
$-3x^2$	$-4x$	-2
$-x^2$	$4x$	3

Multiplication

Similar Variable: In multiplication of single terms having same variable, coefficients of the terms are multiplied and the exponents are added; for example $3x^3 \times 5x^4 = 15x^7$

Dissimilar Variable:

In multiplication of two algebraic expressions, multiply each term of first expression with all terms of the other one by one. Combine like terms of the resultant terms and write all these terms as an expression; for example $3K^3 \times 2P^3 = 6K^3P^3$

Factors

If an algebraic expression is a product of other algebraic expressions then these expressions are called factors of the original expression for example,

$18x^4y + 12x^2y = 6x^2y\ (3x^2 + 2)$, Since the product of $6x^2y\ (3x^2 + 2)$ is $18x^4y + 12x^2y$, therefore, $6x^2y$ and $3x^2 + 2$ are factors of $18x^4y + 12x^2y$.

Division of Algebraic Expressions

Single term Expressions: If you are to divide $15x^6$ by $3x^2$, then divide the coefficient of numerator ($15x^6$) by the coefficient of divisor ($3x^2$) that is $15 \div 3 = 5$ which is the coefficient of the resultant expression. Now for dividing x^6 by x^2, just subtract the exponents that is $x^{6-2} = x^4$. The result of this division is $5x^4$.

Multi Term Expressions (Polynomials): If, for example you are to divide $6x^4 + 18x^2 - 24x$ by $3x$, then divide each term of the expression by $3x$ using the same rule described above for single term expression. You can write $\frac{6x^4}{3x} + \frac{18x^2}{3x} - \frac{24x}{3x}$ to have the required expression i.e., $2x^3 + 6x + 8$.

Note

Since algebraic expressions can be multiplied, they can be squared, cubed or raised to any power.

For squaring a term, take the square of the coefficient and multiply the exponent with 2; for example square of $5x^3$ is $25x^6$. Similarly, cube of $5x^4$ is $5^3x^{3\times4} = 125x^{12}$.

The multiplication addition and subtraction of algebraic expressions are commutative, i.e., $2x^3 \times 3y^2 = 3y^2 \times 2x^3 = 6x^3y^2$.

Value of an Expression

If we are provided with the values of all variables used in an expression, we can find the value of the expression. For example, the value of $5x^2 + 2x$ at $x = 2$ is $5 \times 2 + 2 \times 2 = 10 + 4 = 14$

Factoring an Algebraic Expression

Common factor of all terms: If an algebraic expression has a common multiplier in all terms then you can take it as a common factor. For example; $4x^3 + 2xy$ has $2x$ as a common factor i.e., $4x^3 + 2xy = 2x(2x^2 + y)$

Factor a Common Divisor: If an expression is consisted of the difference of two whole squares e.g. $a^2 - b^2$ then factors of this expression will be $(a-b)$, $(a + b)$ i.e., $a^2 - b^2 = (a + b)(a - b)$. For example: $4x^2 - 9y^2$ has factors $(2x - 3y)$ and $(2x + 3y)$

Facts:

- $a^2 + b^2 + 2ab = (a+ b)\ (a+ b) = (a + b)^2$
- $a^2 + b^2 - 2ab = (a - b)\ (a - b) = (a - b)^2$

Exponential Functions

A function is a definition. For example, function y is defined as $y = 5x^2 + 3$. In mathematics function is usually represented as f(x). Usually a variety of symbols are used to represent a function on the GAT, like Ø, ¥, Δ, Ψ, #, @, $, &, *, ., and Λ etc. Don't get panic about GAT quantitative, no technical or complicated mathematical expressions or formulae are involved in solving the questions on the GAT.

Example:

If ¥N = 3N - 1, then find ¥5.

Solution:

Replace N by 5 in the expression. i.e., ¥5 = 3×5 – 1 = 14

Absolute Value:

The absolute value of a number x, denoted by $|x|$, is defined by the formula $|\pm x| = x$. In other words $|x|$ is always positive.

Note:

$|x| = 0$ if and only if $x = 0$

A number and its negative have the same absolute value $|-a| = |a|$

The absolute value of a product is the product of the absolute values $|ab| = |a|\,|b|$

The absolute value of a quotient is the quotient of the absolute value $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

The absolute value of the sum of two numbers is less than or equal to the sum of their absolute values $|a+b| \leq |a| + |b|$

EQUATIONS

An equation is a statement that says that two given algebraic expressions are equal e.g. $3X - 5 = 7$, $x^2 + 2x - 6 = 0$, $x^2 + y^2 = 16$ etc.

- Addition of same number on both sides does not affect the equations.
- Same number can be subtracted from both sides.
- Both sides if multiplied or divided by same number (except 0) do not affect the equation.
- Both sides can be squared.
- Square root of both sides van be taken if both sides are positive.
- Reciprocal of both sides can be taken.

Solution of Equation

Solution of an equation is to find the value of the variable used in the equation. For example, $3X - 5 = 7$ can be solved to find the value of x.

3X - 5 = 7 → 3x = 7 + 5 → x = $\frac{12}{3}$ → x = 4 is the solution of the equation.

Solution of equation involving one variable

Step	Action	Example
1	Remove fractions by multiplying both sides with least common denominator	Multiply both side of 3(X-2) + 2(x +1) + $\frac{1}{2}$ with 2 to get 6(x-2) = 4(x + 1) + 1
2	Remove parenthesis and use the rule a (b +c) = ab + ac	6x – 12 = 4x + 4 + 1
3	Combine like terms on both sides	6x -12 = 4x + 5
4	By adding and subtracting get all variables on left side	Subtract 4x from both sides 2x – 12 = 5
5	By adding and subtracting get constants on right side	Add 12 on both sides 2x = 17
6	Divide both sides by the coefficient of variable	Divide both sides by 2 and get x = 17/2

Solution of Quadratic Equations

Quadratic equation is a second – degree equation (highest power of variable is 2) of form $ax^2 + bx + c = 0$ where a, b and c are real numbers, for example $3x^2 + 5x + 2 = 0$

Quadratic equation has two solutions: $x = \frac{-b+\sqrt{b2-4ac}}{2a}$ and $x = \frac{-b-\sqrt{b2-4ac}}{2a}$

Normally, quadratic equation has two real values for the variable, but it may have only one solution if the discriminator $(b^2 - 4ac) = 0$. In this case solution of equation will be $x = \frac{-b}{2a}$

Alert

If a quadratic equation has fractions then it is better to remove these fractions by multiplying all terms by LCD.

Quadratic equations having only one solution are always a complete square.

If $b^2 - 4ac < 0$ then equation has no real solution because square root of a negative number is not real.

Maximum number of solutions of an equation is equal to its degree (highest power of variable).

SYSTEM OF EQUATIONS

A group of equations having more than one variable is called simultaneous equations or system equations.

Example: 2x + 3y = 23 and 4x – 2y = 6

Solution of simultaneous equations

The solution or solution set of system of equations consisted of value of all the variables present in the equations. It is necessary that these values must satisfy all the equations.

Solution of simultaneous equations

From the first equation, find the value of one variable in terms of the other variable.

Put the value of the variable, found in the first step, into the second equation. The second equation will be converted to a single variable equation.

Find the value of the variable in the converted equation.

Find the value of the second variable by putting the value of the second variable found in the third step.

Example:

Solve the simultaneous equation $2x + 3y = 23$ and $4x - 2y = 6$.

Solution:

To solve the equations is to find the values of x and y.

Find the value of first variable, x from the first equation in terms of the second variable, y.

$$2x + 3y = 23$$

$$2x = 23 - 3y$$

$$x = \frac{23 - 3y}{2}$$

Put the value of x into the second equation

$$4\left(\frac{23 - 3y}{2}\right) - 2y = 6$$

$$2(23 - 3y) - 2y = 6$$

$$46 - 6y - 2y = 6$$

$$-8y = 6 - 46$$

$$-8y = -40$$

$$Y = \frac{40}{8} = 5$$

Put the value of $y = 5$ in the first equation

$$2x + 3(5) = 23$$

$$2x + 15 = 23$$

$$2x = 8$$

$$X = 4$$

The solution set: $x = 4$ and $y = 5$

The system of three equations involving three variables can be solved by similar method i.e., first find two variable as described above and then putting the values of two variables in the third equation to find the third variable.

Note:

You need one equation to solve single variable equation.

You need two equations to solve two variable system of equations.

You need n equations to solve n variable system of equations.

MATRICES AND DETERMINANTS

A **matrix** is simply a set of numbers arranged in a rectangular table.

On the right is an example of a 2 × 4 matrix. It has 2 rows and 4 columns. We usually write matrices inside parentheses ( ) or brackets [ ].

$$\begin{bmatrix} 3 & 1 & 5 \\ 7 & 2 & 0 \end{bmatrix}$$

We can add, subtract and multiply matrices together, under certain conditions.

A **determinant** is a square array of numbers (written within a pair of vertical lines) which represents a certain sum of products. A determinant of a matrix represents a single number. We obtain this value by multiplying and adding its elements in a special way. We can use the determinant of a matrix to solve a system of simultaneous equations.

For example, if we have the (square) 2 × 2 matrix:

$$\begin{bmatrix} 5 & 7 \\ 2 & -3 \end{bmatrix}$$

then the **determinant** of this matrix is written within vertical lines as follows:

$$\begin{vmatrix} 5 & 7 \\ 2 & -3 \end{vmatrix}$$

Calculating a 2 × 2 Determinant

In general, we find the value of a 2 × 2 determinant with elements *a, b, c, d* as follows:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \text{ad} - \text{bc}$$

We multiply the diagonals (top left × bottom right first), then subtract.

Example:

$$\begin{vmatrix} 4 & 1 \\ 2 & 3 \end{vmatrix} = 4 \times 3 - 1 \times 2 = 10$$

The final result is a single number.

INEQUALITIES

Inequalities represent relation of two algebraic expressions in which one expression is greater or less than the other expression. For example: $x < 3$, $2x + 7 < 15$, $x + y > 7$

The solution of an inequality is not a single value rather it is a set of all values satisfying the inequality, for example; $x < 3$ means all numbers less than three contribute the solution set.

Addition or subtraction of same number on both sides does not affect the inequality.

If both sides are multiplied or divided by same positive number (except 0) inequality remains the same.

Both sides can be squared.

Square root of both sides can be taken if both sides are positive.

Multiplying or dividing both sides with a negative number reverses the inequality.

Taking reciprocal of both sides reverses the inequality.

All methods of solving equations can be used to solve inequalities.

SOLVED EXERCISE

Directions: Each passage in this group is followed by questions based on its content. After reading a passage, choose the best answer to each question. Answer all questions following a passage on the basis of what is stated or implied in that passage.

The answers and explanations of the questions have been given at the bottom of each question.

1. If 2x + 5y = 18 and x = 4 then what is the value of y?

A. 2 B. 3
C. 4 D. 5

Explanation:

A is the best response.

Put the value of x in the equation.

$2(4) + 5y = 18$

$5y = 18 - 8$

$Y = \frac{10}{5} = 2$

2. In All Pakistan Weight Lifting competition in Multan, double of first lift of Mr. Sahiwal was 150 kilograms more than his second lift and the sum of two lifts was 375 kilograms. What was the weight of his first lift?

A. 300 B. 350
C. 175 D. 100

Explanation:

C is the best response.

L is the weight of first lift and P is the weight of the second lift. Based on given conditions you have the equation 2L = P + 150 and L + P = 375

3. ^ of N is defined as five subtracted from twice the N. What is the value of ^7.

A. 7 B. 5
C. 14 D. 9

Explanation:

D is the best response.

Given function is ^N = 2N – 5

So, ^7 = 2×7 – 5 = 14 – 5 = 9

4. If three times of a number is 6 more than twice the number. What is the value of the number?

A. 2 B. 4
C. 6 D. 8

Explanation:

C is the best response.

X is the number. Based on the given condition, the equation is 3x = 2x + 6

$3x - 2x = 6$

$X = 6$

5. Saima bought two black pencils and three red pencils for Rs. 23. After a week, she bought three black pencils and two red pencils for Rs. 16. If price of pencils remained the same for both transactions, how much she paid for one black pencil?

A. 2 B. 3
C. 4 D. 5

Explanation:

A is the best response.

If B is the price of black pencil and R is the price of red pencil, then First equation based on the information given is $2B + 3R = 23$ and the second equation is $3B + 2R = 16$.

6. The value of $x^2 + 5x + 6$ at $x = 2$ is?

A. 2 B. 20
C. 40 D. 10
E. 15

Explanation

By putting the value of $x = 2$ in the equation, you have $2^2 + 5\times2 + 6 = 4 + 10 + 6 = 20$. The right answer is B.

7. One positive number is $\frac{2}{3}$ of the other and their product is 24.what is the sum of the two?

A. 6 B. 18
C. 36 D. 24
E. 10

Explanation

E is the best response.

Convert the given conditions in equations,

$X = \frac{2}{3}Y$ and $XY = 24$. Solve the equations to have the answer.

8. If $x + 2y = 11$ and $2x + 3y = 17$ then y is?

A. 6 B. 5
C. 4 D. 3
E. 0

Explanation

B is the best response.

Multiply the first equation by 2 and then subtract the second equation from it.

9. Twice the age of son is 4 year more than the age of his father. What is the age of the son if father is of 40?

A. 22 B. 20
C. 18 D. 16
E. 14

Explanation

A is the best response.

10. The equation based on given conditions is $2S - 4 = F$. Now put $F = 40$.

If $\#n = (n-5)^2 + 5$, then find $\#3 \times \#5$.

A. 50
B. 55
C. 30
D. 40
E. 45

Explanation

E is the best response.

$\#3 = (3 - 5)^2 + 5 = 9$

$\#5 = (5 - 5)^2 + 5 = 5$

$\#3 \times \#5 = 9 \times 5 = 45$